

An off-shell D-brane action at order α'^2 in flat spacetime

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Abstract

We use compatibility of the second fundamental form corrections to DBI action at order α'^2 which includes trace of the second fundamental form, with T-duality and with the linear S-duality as guiding principles to find an off-shell D-brane action at order α'^2 in type II superstring theories in flat spacetime.

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1 Introduction and Results

In flat spacetime with no massless closed string background, effective action of a D_p -brane in type II superstring theories at long wavelength limit is given by Dirac-Born-Infeld (DBI) action [1, 2]

$$S_p = -T_p \int d^{p+1} \sigma \sqrt{-\det(\tilde{G}_{ab} + F_{ab})} \quad (1)$$

where we normalize the gauge field to absorb the factor of $2\pi\alpha'$ in front of the gauge field strength F_{ab} which usually appears in the literature. In above equation, \tilde{G}_{ab} is the pull-back of the bulk flat metric onto the world-volume², *i.e.*,

$$\begin{aligned} \tilde{G}_{ab} &= \frac{\partial X^\mu}{\partial \sigma^a} \frac{\partial X^\nu}{\partial \sigma^b} \eta_{\mu\nu} \\ &= \eta_{ab} + \partial_a \chi^i \partial_b \chi^j \eta_{ij} \end{aligned} \quad (2)$$

where in the second line the pull-back is in the static gauge. The DBI action (1) is invariant under T-duality and its equations of motion for the case of $p = 3$, are invariant under S-duality [3, 4, 5]. With our normalization for the gauge field, the DBI action is at the leading order of α' . The first correction to this action is at order α'^2 in which we are interested in this paper. The α' corrections to Born-Infeld action have been studied in [6, 7, 8, 9, 10, 11] in the σ -model approach.

For zero gauge field, the general covariance requires the world-volume couplings at any order of α' consists of various contractions of the second fundamental form Ω and its derivatives. At order α'^2 , such couplings have structures Ω^4 or $(D\Omega)^2$. The latter couplings are not consistent with supersymmetry [12], and the former couplings have been found in [12, 13] through the curvature squared couplings to be

$$\begin{aligned} S_1 = & -\frac{\pi^2 \alpha'^2 T_p}{48} \int d^{p+1} \sigma \sqrt{-\det(\tilde{G}_{ab} + F_{ab})} \left[4\Omega^{abi} \Omega_{ab}{}^j \Omega_{cd}{}^i \Omega_{cdj} - 4\Omega^{abi} \Omega_a{}^c{}_i \Omega_b{}^{dj} \Omega_{cdj} \right. \\ & \left. + 4\Omega_a{}^i \Omega^{bc}{}_i \Omega_b{}^{dj} \Omega_{cdj} - 6\Omega_a{}^i \Omega_b{}^{j} \Omega_{cd}{}^i \Omega_{cdj} + 2\Omega_a{}^i \Omega^b{}_{bi} \Omega_c{}^j \Omega_{dj}^c \right] \end{aligned} \quad (3)$$

The second fundamental form in flat spacetime reduces to the acceleration (see *e.g.*, [18]), *i.e.*, $\Omega_{ab}{}^i = \partial_a \partial_b \chi^i$. The world-volume indices in above action are raised by the inverse of the pull-back metric, \tilde{G}^{ab} , and the transverse indices are lowered by

$$\tilde{\perp}_{ij} = \eta_{ij} - \eta_{ik} \eta_{jl} \partial_a \chi^k \partial_b \chi^l \tilde{G}^{ab} \quad (4)$$

The couplings (3) are consistent with the S-matrix element of four transverse scalar fields at order α'^2 [12], however, they include the trace of the second fundamental form which is zero

²Our index convention is that the Greek letters (μ, ν, \dots) are the indices of the space-time coordinates, the Latin letters (a, b, c, \dots) are the world-volume indices and the letters (i, j, k, \dots) are the normal bundle indices. The Killing index in the reduction of 10-dimensional spacetime to 9-dimensional spacetime is y .

on-shell. They have been found in [13] by studying the consistency of the couplings of one massless closed and two open strings under T-duality and linear S-duality transformations. The action (3) contains couplings at all orders of the scalar fields.

It is known that the S-matrix elements of four massless NS states have no massless pole at order α'^2 . On the other hand, the S-matrix elements satisfy the Ward identity corresponding to the S-duality and T-duality [19], *i.e.*, the amplitudes are invariant under linear dualities. The T-duality transformations for the massless NS states are linear whereas the S-duality transformation for the gauge field is nonlinear [3]. One may expect then the four off-shell NS couplings at order α'^2 to be invariant under T-duality and/or linear S-duality. We are going to impose these constraints to include four non-constant gauge field strengths into the action (3). The T-duality constraint on higher derivative couplings of branes have been studied in [14, 15, 16]. Four-field on-shell couplings to all orders in α' has been obtained in [17] from the corresponding S-matrix element.

Four-derivative on-shell corrections to the DBI action involving four gauge field strengths and four derivatives have been known for a long time [8]. They do not include the couplings which are related by T-duality to the off-shell couplings in (3). To include these terms, one may constrain the couplings to be invariant under the dualities, reduces to (3) when the gauge fields are zero, and to be consistent with the corresponding S-matrix element in string theory. There are two ways to impose the dualities. One may first consider all S-duality invariant couplings with unknown coefficients and then imposes the T-duality constraint and the reduction to (3) to find the coefficients. Or, one may consider all arbitrary couplings with unknown coefficients and imposes the T-duality constraint, the reduction to (3) and then the S-duality constraint. It turns out that the two results are not identical. In fact, it turns out that in the latter approach the S-duality can be imposed only on on-shell couplings. In other words, the T-duality and the linear S-duality do not commute. However, the couplings in both cases reproduce the same S-matrix element, so they are identical up to a field redefinition.

In the first approach, one has to consider all linear S-duality invariant couplings. For two gauge fields, one may consider the tensor Q_{abcdef} which is defined by the following expression:

$$Q_{abcdef} \equiv \partial_a F_{bc} \partial_d F_{ef} + \partial_a (*F)_{bc} \partial_d (*F)_{ef} \quad (5)$$

where $(*F)_{ab} = \epsilon_{abcd} F^{cd}/2$. It is invariant under the linear S-duality for D₃-brane. To extend the couplings involving this tensor to the arbitrary D_p-brane, one has to replace the contraction of two four-dimensional Levi-Civita tensors in terms of metric to produce the corresponding couplings on the world-volume of D₃-brane, and then extend the couplings to the arbitrary dimensions. For four gauge fields, however, the S-duality invariant expression which includes two Q 's, involves four Levi-Civita tensors. There are three different pairings of these tensors. In section 3, we will show that the different pairings produce different expressions for contractions of four gauge fields. Apart from this ambiguity, one can consider all contractions of Q and Ω at order α'^2 with unknown coefficients, and impose the T-duality

constraint and the reduction to (3) to find the coefficients. For the specific paring of the Levi-Civita tensors that the two Levi-Civita tensors in each Q contract with each other, we have done the calculation and found that above constraints fix the constants such that the couplings satisfy the S-matrix element without any further constraints on the coefficients. This approach can not be extended to more than four-field couplings because the contact terms of the corresponding S-matrix elements do not satisfy the S-dual Ward identity (see section 3).

In the second approach, one has to consider all contractions of Ω and ∂F at order α'^2 with unknown coefficients and constrain them to be consistent with the T-duality and with the four-field couplings in (3). Imposing these constraints, one finds that the resulting off-shell couplings do not satisfy the S-duality constraint even at two gauge field level. So we are forced to impose on-shell S-duality. On the other hand, the 4-point S-matrix element satisfy the S-dual Ward identity [20], so to impose the on-shell S-duality, one may impose the consistency of the couplings with S-matrix element. This gives one extra constraint. Up to some total derivative terms and the Bianchi identity, we have found the following couplings between two scalar fields and two gauge fields:

$$S_2 = -\frac{\pi^2 \alpha'^2 T_p}{48} \int d^{p+1} \sigma \sqrt{-\det(\tilde{G}_{ab} + F_{ab})} \left[4\Omega_{cd}^i \Omega_{ei}^e \partial_a F_b^d \partial^a F^{bc} - 8\Omega_c^{ei} \Omega_{dei} \partial_a F_b^d \partial^a F^{bc} \right. \\ \left. + 12\Omega_c^{ei} \Omega_{dei} \partial^a F^{bc} \partial_b F_a^d - 4\Omega_{cd}^i \Omega_{ei}^e \partial^a F^{bc} \partial_b F_a^d - 4\Omega_d^i \Omega_{ei}^e \partial^a F_a^b \partial^c F_{bc} \right. \\ \left. - 4\Omega_c^{ei} \Omega_{dei} \partial^a F_a^b \partial^c F_b^d + 12\Omega_{cd}^i \Omega_{ei}^e \partial^a F_a^b \partial^c F_b^d + 8\Omega_{ac}^i \Omega_{dei} \partial^a F^{bc} \partial^d F_b^e \right] \quad (6)$$

The world-volume indices in (6) are raised by η^{ab} , and the transverse indices are lowered by η_{ij} . The above constraints also fix the following couplings between four gauge fields:

$$S_3 = -\frac{\pi^2 \alpha'^2 T_p}{48} \int d^{p+1} \sigma \sqrt{-\det(\tilde{G}_{ab} + F_{ab})} \left[2\partial^a F^{bc} \partial_b F_a^d \partial_c F^{ef} \partial_d F_{ef} \right. \\ \left. - \frac{1}{2} \partial_a F_{bc} \partial^a F^{bc} \partial_d F_{ef} \partial^d F^{ef} - 2\partial^a F_a^b \partial^c F_{bc} \partial_d F_{ef} \partial^d F^{ef} - 4\partial^a F^{bc} \partial_b F_c^d \partial_d F^{ef} \partial_e F_{af} \right. \\ \left. + 2\partial_a F^{de} \partial^a F^{bc} \partial_f F_{ce} \partial^f F_{bd} + 2\partial^a F_a^b \partial^c F_b^d \partial_d F_c^e \partial^f F_{ef} + 6\partial^a F_a^b \partial_c F^{ef} \partial^c F_b^d \partial_d F_{ef} \right] \quad (7)$$

The couplings (6) and (7) are the T-dual completion of four scalar couplings in (3) and are consistent with on-shell linear S-duality. The above actions are consistent with the S-matrix elements of four massless NS vertex operators, however, there are many couplings in these actions that are zero on-shell. They have been found by the duality constraints. Unlike the first approach, this approach can be extended to higher order fields. It would be interesting then to find the T-dual completion of all infinite scalar fields in (3).

2 Calculations

In this section, using the Mathematica package “xAct” [21], we are going to write all four-field couplings of gauge field and/or transverses scalar fields with unknown coefficients. We then constrain the coefficients by imposing the consistency of the couplings with the dualities. Since the known scalar couplings (3) involve only second derivative of the scalar fields and T-duality transforms the gauge field along the Killing direction to the scalar field in the dual theory, *i.e.*, $A_y \rightarrow \chi^y$, we expect the four gauge field couplings to have structure $(\partial F)^4$ and two gauge field two scalar couplings to have structure $(\partial F)^2 \Omega^2$. In the first approach, one first uses the S-duality constraint and then the T-duality constraint, whereas in the second approach one first uses the T-duality and then the on-shell S-duality constraint. The calculations in both cases are similar so in this section we illustrate how we got the results in (6) and (7) in the second approach. In section 3, we discuss the couplings in the first approach.

So in the second approach, we have to consider all contractions of Ω and $\psi_{abc} \equiv \partial_a F_{bc}$ at order α'^2 . Using “xAct”, one finds the following 65 different contractions:

$$\begin{aligned}
S = & -\frac{\pi^2 T_p \alpha'^2}{48} \int d^{p+1} \sigma \sqrt{-\det(\tilde{G}_{ab} + F_{ab})} \left[c_1 \psi_a^{de} \psi^{abc} \psi_{bd}^f \psi_{cef} + c_2 \psi_a^{de} \psi^{abc} \psi_{bc}^f \psi_{def} \right. \\
& + c_3 \psi_{ab}^d \psi^{abc} \psi_c^{ef} \psi_{def} + c_4 \psi^{abc} \psi_{ba}^d \psi_c^{ef} \psi_{def} + c_5 \psi_a^a \psi_c^{ef} \psi_b^c \psi_{def} + c_6 \psi_{abc} \psi^{abc} \psi_{def} \psi^{def} \\
& + c_7 \psi_a^a \psi_b^c \psi_{bc}^d \psi_{def} \psi^{def} + c_8 \psi^{abc} \psi_b^{de} \psi_{cd}^f \psi_{eaf} + c_9 \psi^{abc} \psi_{bc}^d \psi_d^{ef} \psi_{eaf} + c_{10} \psi_a^{de} \psi^{abc} \psi_{bd}^f \psi_{ecf} \\
& + c_{11} \psi_a^a \psi_b^c \psi_c^d \psi_d^{ef} \psi_{ecf} + c_{12} \psi_{ab}^d \psi^{abc} \psi_c^{ef} \psi_{edf} + c_{13} \psi^{abc} \psi_{ba}^d \psi_c^{ef} \psi_{edf} + c_{14} \psi_a^a \psi_b^c \psi_{bc}^d \psi_c^{ef} \psi_{edf} \\
& + c_{15} \psi_a^a \psi_b^c \psi_c^{ef} \psi_b^d \psi_{edf} + c_{16} \psi_{abc} \psi^{abc} \psi^{def} \psi_{edf} + c_{17} \psi^{abc} \psi_{bac} \psi^{def} \psi_{edf} + c_{18} \psi_a^a \psi_b^c \psi_{bc}^d \psi^{def} \psi_{edf} \\
& + c_{19} \psi_{ab}^d \psi^{abc} \psi_{edf} \psi_c^e \psi_{ef} + c_{20} \psi_a^a \psi_b^c \psi_c^d \psi_{edf} \psi_e^e \psi_{ef} + c_{21} \psi^{abc} \psi_b^{de} \psi_{dc}^f \psi_{fae} + c_{22} \psi_a^{de} \psi^{abc} \psi_{bd}^f \psi_{fce} \\
& + c_{23} \psi_a^{de} \psi^{abc} \psi_{bc}^f \psi_{fde} + c_{24} \psi_{ab}^d \psi^{abc} \psi_c^e \psi_{fde} + c_{25} \psi^{abc} \psi_{ba}^d \psi_c^e \psi_{fde} + c_{26} \psi_a^a \psi_b^c \psi_c^d \psi_e^e \psi_{fde} \\
& + c_{27} \psi_a^{de} \psi^{abc} \psi_{fde} \psi_{bc}^f + c_{28} \psi_a^{de} \psi^{abc} \psi_{fce} \psi_{bd}^f + c_{29} \psi_a^a \psi_b^c \psi_{cd}^e \psi_{ef}^e + c_{30} \psi_a^a \psi_b^c \psi_{cd}^e \psi_b^e \psi_{ef}^e \\
& + c_{31} \psi_a^a \psi_b^c \psi_c^d \psi_{dc}^e \psi_{ef}^e + c_{32} \psi_a^a \psi_b^c \psi_{bc}^d \psi_d^e \psi_{ef}^e + c_{33} \psi_a^a \psi_b^c \psi_{cd}^e \psi_{ef}^e + c_{34} \psi^{abc} \psi_b^d \psi_{ae}^i \Omega_{cdi} \\
& + c_{35} \Omega_a^{cj} \Omega^{abi} \Omega_b^d \Omega_{cdi} + c_{36} \Omega_a^c \Omega^{abi} \Omega_b^{dj} \Omega_{cdj} + c_{37} \Omega_a^a \Omega_b^{dj} \Omega^{bc} \Omega_{cdj} + c_{38} \psi^{abc} \psi_b^{de} \Omega_{ad}^i \Omega_{cei} \\
& + c_{39} \psi^{abc} \psi_b^d \Omega_{ad}^i \Omega_{cei} + c_{40} \psi_a^{de} \psi^{abc} \Omega_{bd}^i \Omega_{cei} + c_{41} \psi_a^a \psi_b^{cde} \Omega_{bd}^i \Omega_{cei} + c_{42} \Omega_{ab}^j \Omega^{abi} \Omega_{cdj} \Omega_{cd}^i \\
& + c_{43} \Omega_a^a \Omega_b^{bj} \Omega_{cdj} \Omega_{cd}^i + c_{44} \Omega_{abi} \Omega^{abi} \Omega_{cdj} \Omega_{cd}^j + c_{45} \Omega_a^a \Omega_b^{bi} \Omega_{cdj} \Omega_{cd}^j + c_{46} \psi^{abc} \psi_b^d \Omega_{ac}^i \Omega_{dei} \\
& + c_{47} \psi^{abc} \psi_{bc}^d \Omega_{ac}^i \Omega_{dei} + c_{48} \psi^{abc} \psi_{bc}^d \Omega_{ac}^i \Omega_{dei} + c_{49} \psi_a^a \psi_b^c \Omega_{bc}^d \Omega_{dei} + c_{50} \psi_{ab}^d \psi^{abc} \Omega_c^e \Omega_{dei} \\
& + c_{51} \psi^{abc} \psi_{ba}^d \Omega_c^e \Omega_{dei} + c_{52} \psi_a^a \psi_b^c \Omega_c^d \Omega_{dei} + c_{53} \Omega_a^a \Omega_b^{bi} \Omega_c^j \Omega_{cd}^j + c_{54} \psi_{abc} \psi^{abc} \Omega_{dei} \Omega_{dei} \\
& + c_{55} \psi^{abc} \psi_{bac} \Omega_{dei} \Omega_{dei} + c_{56} \psi_a^a \psi_b^c \Omega_{bc}^d \Omega_{dei} \Omega_{dei} + c_{57} \psi^{abc} \psi_{bc}^d \Omega_{ad}^i \Omega_{ei}^e + c_{58} \psi^{abc} \psi_{bc}^d \Omega_{ad}^i \Omega_{ei}^e \\
& + c_{59} \psi_a^a \psi_b^c \Omega_{bd}^i \Omega_{ei}^e + c_{60} \psi_{ab}^d \psi^{abc} \Omega_{cd}^i \Omega_{ei}^e + c_{61} \psi^{abc} \psi_{ba}^d \Omega_{cd}^i \Omega_{ei}^e + c_{62} \psi_a^a \psi_b^c \Omega_{cd}^i \Omega_{ei}^e \\
& \left. + c_{63} \psi_{abc} \psi^{abc} \Omega_{d}^d \Omega_{ei}^e + c_{64} \psi^{abc} \psi_{bac} \Omega_{d}^d \Omega_{ei}^e + c_{65} \psi_a^a \psi_b^c \Omega_{bc}^d \Omega_{d}^d \Omega_{ei}^e \right] \quad (8)
\end{aligned}$$

where c_1, \dots, c_{65} are unknown coefficients. In writing the above couplings we have used the mono-term symmetries of Ω and ψ , *i.e.*, the second fundamental form is symmetric with

respect to its first two indices and ψ is antisymmetric with respect to last two indices. The tensor ψ has also multi-term symmetry, *i.e.*, the Bianchi identity, which is not imposed in (8). Moreover, for a specific relation between the coefficients, some combinations of the above couplings are total derivative terms.

Using the Bianchi identity and ignoring some total derivative terms, one can find some relations between the coefficients in (8). One may try to find such relations and use them to reduce the couplings in (8) to independent ones. Then imposes the T-duality constraint to find the relations between the coefficients of the independent terms. Alternatively, one may first use the T-duality constraint to find relations between the coefficients in (8) and then impose the Bianchi identity and remove the total derivative terms. In this paper, we follow the latter approach which is much easier to do with computer, as we will see shortly.

To constraint the above couplings to be consistent with T-duality, following [14], we reduce the 10-dimensional space-time to the 9-dimensional space-time. It reduces (8) to two different actions S_p^w and S_p^t . In S_p^w , the Killing direction y is a world-volume direction, *i.e.*, $a = (\tilde{a}, y)$ and in S_p^t the Killing direction is a transverse direction, $i = (\tilde{i}, y)$. The transformation of S_p^w under the T-duality which is called S_{p-1}^{wT} , may be equal to S_{p-1}^t up to some total derivative terms which must be ignored in the action, *i.e.*,

$$S_{p-1}^{wT} - S_{p-1}^t = 0 \quad (9)$$

This constrains the unknown coefficients in the Lagrangian (8). Note that if one does not ignore the total derivative terms, then one would find some unnecessary constraints which make the total derivative terms in the p -dimensions to be T-duality invariant. To drop the total derivative terms, we transform the terms in $S_{p-1}^{wT} - S_{p-1}^t$ to the momentum space. This labels fields and their momenta. For the identical fields we have to symmetrize the labels as well, *i.e.*, $\partial^a A^b \partial^c A^d B^e C^f$ transforms to

$$\int d^p p_1 d^p p_2 d^p p_3 d^p p_4 e^{ip_1 \cdot x + ip_2 \cdot x + ip_3 \cdot x + ip_4 \cdot x} \frac{1}{2} (-p_1^a A_1^b p_2^c A_2^d - p_2^a A_2^b p_1^c A_1^d) B_3^e C_4^f \quad (10)$$

The integral $\int d^p x$ in the action, then produces a delta function imposing the conservation of momentum $\delta^p(p_1 + p_2 + p_3 + p_4)$. Using this, one then write p_4 in terms of $-p_1 - p_2 - p_3$. This step drops all terms in $S_{p-1}^{wT} - S_{p-1}^t$ that are total derivatives.

We have found that the above T-duality constraint gives the following 21 equations between the constants:

$$\begin{aligned} c_{41} &= -c_{20} - c_{26} + c_{38} + c_{39} + 2c_{40}, c_{42} = -c_{10} + c_{19} - c_{21} - c_{22} + c_{24} + c_{25} + 2c_{28} - c_{35}, \\ c_{43} &= 2c_{10} + 2c_{21} + 2c_{22} - 4c_{28} + c_{30} + c_{31} + 2c_{35}, c_{46} = -2c_{10} + 2c_{19} - 2c_{21} - 2c_{22} \\ &+ 2c_{24} + 2c_{25} + 4c_{28} - c_{34} - c_{39}, c_{49} = c_{29} + c_{30} - 2c_{33} - \frac{c_{38}}{2} - \frac{c_{39}}{2} - c_{40}, \\ c_5 &= -2c_{10} - \frac{c_{11}}{2} - \frac{c_{15}}{2} - \frac{c_{20}}{2} - 2c_{21} - 2c_{22} + 4c_{28} - 2c_{35} - \frac{c_{37}}{2}, \end{aligned}$$

$$\begin{aligned}
c_{50} &= 2c_{34} - 4c_{35} + 2c_{36} + 2c_{39} + c_{47} - 2c_{48}, c_{51} = c_{12} + c_{13} + 2c_{19} + c_{24} + 2c_3 - 2c_{34} \\
&+ 4c_{35} - 2c_{36} + \frac{c_{38}}{2} - \frac{3c_{39}}{2} + 2c_4 + c_{40} - c_{47} + 2c_{48}, c_{52} = 2c_{34} - 4c_{35} - c_{37} + 2c_{39}, \\
c_{53} &= -\frac{c_{10}}{2} - \frac{c_{21}}{2} - \frac{c_{22}}{2} + c_{28} + c_{32} - \frac{c_{35}}{2}, c_{55} = -\frac{c_{34}}{2} + c_{35} - \frac{c_{39}}{2} + 2c_{44} - 2c_{54}, \\
c_{56} &= -\frac{c_{34}}{2} + c_{35} - \frac{c_{39}}{2} - c_{45}, c_{59} = \frac{c_{38}}{2} + \frac{c_{39}}{2} + c_{40}, c_6 = \frac{c_{10}}{8} - \frac{c_{16}}{2} - \frac{c_{17}}{4} + \frac{c_{21}}{8} + \frac{c_{22}}{8} \\
&- \frac{c_{28}}{4} + \frac{c_{35}}{8} + \frac{c_{44}}{4}, c_{60} = -2c_{34} + 4c_{35} + c_{37} - 2c_{39} + c_{57} - 2c_{58}, c_{61} = -c_{20} - c_{26} + 2c_{34} \\
&- 4c_{35} - c_{37} - \frac{c_{38}}{2} + \frac{3c_{39}}{2} - c_{40} - c_{57} + 2c_{58}, c_{62} = -4c_{10} - 4c_{21} - 4c_{22} + 8c_{28} - 2c_{30} - 2c_{31} \\
&- 2c_{34} - 2c_{39}, c_{64} = \frac{c_{34}}{2} - c_{35} + \frac{c_{39}}{2} + c_{45} - 2c_{63}, c_{65} = c_{10} + c_{21} + c_{22} - 2c_{28} - 2c_{32} + \frac{c_{34}}{2} \\
&+ \frac{c_{39}}{2}, c_7 = \frac{c_{10}}{2} - \frac{c_{18}}{2} + \frac{c_{21}}{2} + \frac{c_{22}}{2} - c_{28} + \frac{c_{35}}{2} - \frac{c_{45}}{2}, c_9 = 2c_{10} + c_{12} + c_{19} + c_2 + 2c_{21} \\
&+ 2c_{22} - 2c_{23} + 4c_{27} - 4c_{28} + 2c_3 + 2c_{35} - c_{36}
\end{aligned} \tag{11}$$

Replacing the above constraints into the action (8), one finds 44 T-dual multiplets.

These 44 multiplets must be reduced to the four scalar couplings in (3). This produces the following 8 equations:

$$\begin{aligned}
c_{28} &= \frac{c_{10}}{2} - \frac{c_{19}}{2} + \frac{c_{21}}{2} + \frac{c_{22}}{2} - \frac{c_{24}}{2} - \frac{c_{25}}{2} + 2, c_{31} = -2c_{19} - 2c_{24} - 2c_{25} - c_{30} + 2, \\
c_{32} &= \frac{c_{19}}{2} + \frac{c_{24}}{2} + \frac{c_{25}}{2}, c_{35} = 0, c_{36} = -4, c_{37} = 4, c_{44} = 0, c_{45} = 0
\end{aligned} \tag{12}$$

Replacing these constraints into the 44 multiplets, one finds 36 T-dual multiplets which reduce to the four scalar couplings in (3).

Now we have to impose the consistency with the linear S-duality. There is no ambiguity in imposing the S-duality constraint on two gauge fields, *i.e.*, one has to replace F with $*F$ and then replace the two Levi-Civita tensors in terms of metric. The result must be identical to the original couplings. We have found this produces constraints which are not consistent with the constraints in (11) and (12). That means the S-duality of equations of motion can not be prompted to the action level in this case. So we impose the linear S-duality on on-shell couplings. On the other hand, the on-shell couplings must be identical to the contact terms of the corresponding S-matrix element. It has been shown in [20] that the S-matrix element of four NS vertex operators satisfy linear T-duality. So to impose the on-shell linear S-duality, one may impose the consistency of the above 36 T-dual multiplets with the S-matrix element. This produces the following constraint: (see Appendix)

$$c_4 = -\frac{c_{12}}{2} - \frac{c_{13}}{2} - c_{19} - \frac{c_{24}}{2} - c_3 + 2 \tag{13}$$

Replacing this constraint into the above 36 multiplets, one finds one multiplet with specific coefficients, *i.e.*, the couplings in (3), (6) and (7), and 35 multiplets with unknown overall coefficients.

Now we are going to show that, after using the Bianchi identity and dropping total derivative terms, all the 35 multiplets with unknown coefficients are reduced to 2 unphysical multiplets. To impose the Bianchi identity, we write the gauge field strength $\partial_a F_{bc}$ in terms of the gauge field, *i.e.*, $\partial_a \partial_b A_c - \partial_a \partial_c A_b$. This makes 23 multiplets with the coefficients $c_1, c_2, c_3, c_{12}, c_{13}, c_{11}, c_{16}, c_{17}, c_{18}, c_8, c_{27}, c_{10}, c_{14}, c_{15}, c_{22}, c_{48}, c_{47}, c_{54}, c_{57}, c_{58}, c_{63}, c_{21}, c_{23}$ to be zero. That means, these constants are the coefficients of terms that are zero by the Bianchi identity. Therefore, it is safe to set these 23 coefficients to zero. Moreover, after using the Bianchi identity, the terms with coefficients $c_{26}, c_{20}, c_{29}, c_{30}, c_{33}$ appears only as $(c_{20} + c_{26})$ and $(c_{29} + c_{30} - 2c_{33})$. That means there are only two independent coefficients. So we set c_{26}, c_{30}, c_{33} to zero.

The terms with coefficients $c_{39}, c_{40}, c_{38}, c_{25}, c_{19}, c_{34}, c_{24}$ are total derivative terms. So these 7 coefficients can also be set to zero. Therefore, there are only 2 independent extra multiplets which are

$$\begin{aligned} & -c_{20}\Omega_{cd}^i\Omega_{ei}^e\partial^a F^{bc}\partial_b F_a^d + c_{29}\Omega_b^{ei}\Omega_{dei}\partial^a F_a^b\partial^c F_c^d - c_{20}\Omega_{bd}^i\Omega_{cei}\partial^a F_a^b\partial^c F^{de} \\ & -\frac{1}{2}c_{20}\partial^a F_a^b\partial_c F^{ef}\partial^c F_b^d\partial_d F_{ef} + c_{20}\partial^a F_a^b\partial^c F_b^d\partial_e F_{df}\partial^e F_c^f + c_{29}\partial^a F_a^b\partial_b F^{cd}\partial_c F_d^e\partial^f F_{ef} \end{aligned} \quad (14)$$

As can be seen, these multiplets have no couplings of four transverse scalar fields, so they are not related to the couplings in (3). It is also obvious that the 4-point S-matrix element of above couplings are zero, because they involve either the trace of the second fundamental form or $\partial^a F_a^b$ which are zero on-shell.

We speculate that the above multiplets do not produce any other S-matrix element. To give one more example, we calculate the S-matrix element of two scalars, one gauge field and one B-field. This amplitude is given by the following Feynman rule:

$$\mathcal{A} = V(B_4, A)G(A)V(A, A_3, \chi_2, \chi_1) + V(B_4, A_3, \chi_2, \chi_1) \quad (15)$$

where χ_1, χ_2, A_3, B_4 are the polarizations of the external fields. The symmetry of string theory under the gauge transformation $A_a \rightarrow A_a - \Lambda_a$ requires one to extend the gauge field strength in effective actions to $F + B$. Using this replacement in (14), one can calculate the vertices $V(B_4, A_3, \chi_2, \chi_1)$ and $V(A, A_3, \chi_2, \chi_1)$, and the vertex $V(B_4, A)$ and propagator $G(A)$ can be calculated from the DBI action. We have done this calculations in details and found zero result. We expect similar result for any other S-matrix element. Therefore, the multiplets (14) are unphysical multiplets. This ends our illustration that all 35 T-dual multiplets are either unphysical or they become total derivatives after using the Bianchi identity.

3 Discussion

In this paper, we have shown that the consistency of all four massless NS couplings at order α'^2 with T-duality, on-shell linear S-duality and with the scalar couplings in (3) fixes all couplings up to two unknown coefficients. The multiplet with the known coefficient are given in (6) and (7). These couplings include many terms which are zero on-shell. They may be removed by using appropriate field redefinitions. The field redefinition, however, change the standard form of the T-duality transformations. In the specific field variables in which the T-duality transformation is $A_y \rightarrow \chi^y$, the off-shell terms must be included in the action in order to be invariant under the T-duality. The two multiplets with unknown coefficients are unphysical and do not contribute to S-matrix.

The actions (6) and (7) are invariant under S-duality only at the on-shell level. If one would like to find an action which is invariant under off-shell S-duality, one should first impose S-duality and then T-duality. We have done this calculation and found the following result:

$$\begin{aligned}
S = & -\frac{\pi^2 \alpha'^2 T_p}{48} \int d^{p+1} \sigma \sqrt{-\det(\tilde{G}_{ab} + F_{ab})} \left[-Q^a{}_{bcde} Q_{cb}{}^f{}_{fde} + \frac{1}{2} Q^{abc}{}_{de} Q_{cd}{}^f{}_{eaf} \right. \\
& -\frac{3}{2} Q^{abc}{}_{de} Q_{dc}{}^f{}_{fae} + 8Q^{abc}{}_{bc}{}^d Q_d{}^{ef}{}_{eaf} + \frac{7}{2} Q^a{}_{bc}{}_{bc}{}^d Q_d{}^{ef}{}_{ef} + 2Q^a{}_{bc}{}_{bc}{}^d Q_c{}^e{}_{fde} \\
& +\frac{1}{4} Q^{abc}{}_{de} Q^f{}_{bdfce} - 2Q^c{}_{de}{}^{de} \Omega_{abi} \Omega^{abi} + 2Q^d{}_{bde}{}^{ce} \Omega_a{}^c{}_i \Omega^{abi} - 4Q^d{}_{de}{}^{ce} \Omega_a{}^c{}_i \Omega^{abi} \\
& -2Q^c{}_{de}{}^{de} \Omega_a{}^i \Omega^b{}_{bi} - 2Q^d{}_{bde}{}^{ce} \Omega_a{}^i \Omega^{bc}{}_i - 4Q^d{}_{de}{}^{ce} \Omega_a{}^i \Omega^{bc}{}_i + 4Q_{ab}{}^{e}{}_{cde} \Omega^{abi} \Omega^{cd}{}_i \\
& \left. +4Q^e{}_{aecbd} \Omega^{abi} \Omega^{cd}{}_i \right] \tag{16}
\end{aligned}$$

where Q is given in (5). It is obviously invariant under linear S-duality for D₃-brane. The Levi-Civita tensors in above action should be evaluated for D₃-brane to find the corresponding contractions between Ω and ∂F . Then the result should be extended to arbitrary D_p-brane. We have found that the above couplings are consistent with the 4-point S-matrix element. In this case there is one unphysical multiplet with unknown coefficient. The above action and the couplings in (6), (7) looks very different. However, since both of them satisfy the same S-matrix element, they are related by field redefinitions.

The action (3) includes all infinite number of scalar couplings through the expansion of the inverse of the pull-back metric in the action. So one may expect that the consistency of the couplings with the dualities and with (3) can be extended to all order of gauge fields. The S-duality is non-linear, and there are indications that D₃-brane effective action is invariant under nonlinear S-duality only at the equations of motion level [3, 24], so we do not expect the effective action at higher orders to be consistent with the nonlinear S-duality. In general, we do not expect them to be invariant under linear S-duality even at the on-shell level. To see this, consider the α' expansion of an S-matrix element which can be separated into two

parts. One part includes massless poles and the other part includes contact terms, *i.e.*,

$$A = A_{\text{pole}} + A_{\text{contact}} \quad (17)$$

The linear S-duality transformation which is on the gauge field strength, *i.e.*, $F \rightarrow *F$, may transform A_{pole} to A_{contact} , so the contact terms which produce the effective action, may not satisfy the S-dual Ward identity³. This is not the case for four gauge fields that we have considered in (16) because there is no massless pole for 4-point function. The S-matrix elements of one massless closed string and two NS states also have no massless poles, so one can use the consistency of the couplings with the linear S-duality and T-duality to find the physical couplings [13].

On the other hand, the T-duality transformation which is on gauge field, *i.e.*, $A_y \rightarrow \chi^y$, does not transform A_{pole} to A_{contact} , so the contact terms always satisfy the T-dual Ward identity. As a result, one expects the consistency of the higher order couplings with T-duality to be a valid constraint at higher order fields. Therefore, we expect the consistency of the couplings with the T-duality and with (3) to be extended to all order of gauge fields.

The T-duality may fix the presence of F_{ab} in the pull-back of the flat metric in (3) by extending it to the following expression:

$$\tilde{G}_{ab} \longrightarrow G_{ab} = \eta_{ab} + \partial_a \chi^i \partial_b \chi^j \eta_{ij} - F_{ac} F_{db} \tilde{G}^{cd} \quad (18)$$

The above extension then produces the following extensions for the metrics \tilde{G}^{ab} and $\tilde{\perp}_{ij}$ which appear at various places in (3):

$$\begin{aligned} \tilde{G}^{ab} &\longrightarrow G^{ab} \\ \tilde{\perp}_{ij} &\longrightarrow \perp_{ij} = \eta_{ij} - \eta_{ik} \eta_{jl} \partial_a \chi^k \partial_b \chi^l \hat{G}^{ab} \end{aligned} \quad (19)$$

To verify that the replacement (18) is consistent with T-duality, suppose the D-brane is along the circle on which the T-duality is imposed. One can easily verify that the world-volume indices of G^{ab} in (3) are contracted with the derivatives in Ω_{ab}^i or in $\partial_a \chi^k \partial_b \chi^l$, so they can not be the Killing index y . On the other hand, when a, b are not the Killing index, *i.e.*, $a = \tilde{a}, b = \tilde{b}$, using the prescription given in [14], one can verify that $G^{\tilde{a}\tilde{b}}$ is invariant under T-duality. To the second order of fields, it is

$$\begin{aligned} G^{\tilde{a}\tilde{b}} &= \eta^{\tilde{a}\tilde{b}} - \partial^{\tilde{a}} \chi^i \partial^{\tilde{b}} \chi^j \eta_{ij} + F^{\tilde{a}\tilde{c}} F^{\tilde{d}\tilde{b}} \eta_{\tilde{c}\tilde{d}} + \partial^{\tilde{a}} A_y \partial^{\tilde{b}} A_y \eta^{yy} \\ &\xrightarrow{T} \eta^{ab} - \partial^a \chi^{\tilde{i}} \partial^b \chi^{\tilde{j}} \eta_{\tilde{i}\tilde{j}} + F^{ac} F^{db} \eta_{cd} - \partial^a \chi^y \partial^b \chi^y \eta_{yy} \\ &= \eta^{ab} - \partial^a \chi^i \partial^b \chi^j \eta_{ij} + F^{ac} F^{db} \eta_{cd} = G^{ab} \end{aligned} \quad (20)$$

³ It has been observed in [22] that the combination of massless poles and contact terms of the S-matrix element of six gauge fields at the leading order of α' which is reproduced by the DBI action, is invariant under linear S-duality.

where in the second line we have used the T-duality transformation $A_y \rightarrow \chi^y$. We have checked to the tenth order of fields and found that $G^{\bar{a}\bar{b}}$ is invariant. Therefore, the replacement (19) forced by T-duality, extends the couplings (3) to include the constant gauge field strength to all orders. The resulting action, however, does not include the structure in which the two indices of the gauge field strength contract with the second fundamental form, *i.e.*, the four world volume indices of Ω 's contract with $F_{ab}F_{cd}$ or the eight world volume indices of Ω 's contract with $F_{ab}F_{cd}F_{ef}F_{gh}$. Moreover, the world volume indices of $\Omega^i\Omega^j$'s may contract with $\partial_a\chi_i\partial_b\chi_j$. Since the world volume indices of Ω 's are derivative indices, such couplings are invariant under T-duality only when the derivatives of the gauge field strength are zero. The couplings when the gauge field strength is not constant, may be found by requiring the invariance of 6-field, 8-field, and higher couplings under T-duality. It would be interesting to use these conditions to find an action for D_p -brane which includes all orders of gauge fields and the transverse scalar fields. Such action for D_9 -brane has been obtained in [9] via string σ -model loop calculations using the boundary state operator language.

In the presence of the massless closed string fields, the S-duality is modified to $SL(2, R)$ symmetry. Unlike the transformation for the gauge field which is via its field strength, all other transformations involve only field potential. So linear $SL(2, R)$ transformations for the massless closed string fields do not transform the massless pole, A_{pole} , to the contact terms, A_{contact} , of the S-matrix element. Therefore, we expect the effective actions which are produced by the contact terms of the S-matrix elements, to be invariant under the linear $SL(2, R)$ transformations. There is a subtlety, however, for the S-matrix elements involving B-field. The B-field in the D-brane effective action appears in two ways. Either through its field strength H or through the replacement $F \rightarrow F + B$. The massless poles in the S-matrix which are produced by the gauge fields, should be combined with some of the contact terms which are produced by $F + B$ terms, to be able to rewrite the S-matrix element in terms of H [25, 26]. Then the H -contact terms of the S-matrix element are the H -couplings in the effective action. These contact terms transforms to the RR $dC_{(2)}$ -couplings under the $SL(2, R)$ transformation. On the other hand, the B-field in the effective action which appears as $F + B$, are expected to be invariant under the $S(L, R)$ transformation via the equations of motion as in the DBI case [5]. Apart from the $(F + B)$ -couplings, we expect the D_3 -brane effective action to be invariant under the $SL(2, R)$ transformation.

In general, the effective action of D_p -brane is expected to be invariant under T-duality transformation, *e.g.*, the DBI action is invariant under T-duality. The transformation is linear for the massless NS fields whereas it is non-linear for NSNS fields [12]. The invariance of the curvature squared terms in O-plane action [12] under the non-linear T-duality, fixes all NSNS terms at order α'^2 [23, 27]. It would be interesting to find all NSNS and RR couplings in D-brane action at order α'^2 by requiring their curvature squared terms to be consistent with the nonlinear T-duality and with the $SL(2, R)$ transformations.

Finally, let us mention a subtlety in evaluating four Levi-Civita tensors in terms of metric that we have encountered when we have been founding the couplings in (16). Using (5) to

write the couplings in (16) in terms of ∂F , one finds terms that have four Levi-Civita tensors. They are produced by QQ terms. For example, one encounters with the following term:

$$\frac{1}{16}\psi_{aln}\psi^a_{gh}\psi_{dmi}\psi_{ejk}\epsilon_b^{fmi}\epsilon^{bcgh}\epsilon_{cf}^{jk}\epsilon^{deln} \quad (21)$$

where $\psi_{abc} \equiv \partial_a F_{bc}$. To rewrite this term in terms of various contractions of ψ 's, one has to use the standard identity $\epsilon^{a_1 a_2 a_3 a_4} \epsilon_{b_1 b_2 b_3 b_4} = -\delta_{[b_1}^{a_1} \delta_{b_2}^{a_2} \delta_{b_3}^{a_3} \delta_{b_4}^{a_4]}$. However, there are three ways to pairs the Levi-Civita tensors. One of them is the following:

$$\begin{aligned} \frac{1}{16}\psi_{aln}\psi^a_{gh}\psi_{dmi}\psi_{ejk}(\epsilon_b^{fmi}\epsilon^{bcgh})(\epsilon_{cf}^{jk}\epsilon^{deln}) &= \frac{1}{2}\psi^a{}_f\psi_m{}^{jk}\psi^m{}_f\psi_{ij}{}^k + \psi^a{}_f\psi_m{}^{jk}\psi^m{}_f\psi_{ji}{}^k \\ &- \psi^a{}_{ij}\psi^{afm}\psi_{fi}{}^k\psi_{kmj} - \frac{1}{2}\psi^a{}_{ij}\psi^{afm}\psi_{fm}{}^k\psi_{kij} - \psi_{af}{}^i\psi^{afm}\psi_m{}^j\psi_{kij} + \psi^a{}_f\psi_{mi}{}^j\psi^m{}_f\psi_{jk}{}^i \end{aligned}$$

where we have used the standard identity in the Levi-Civita tensors in each parenthesis. Another paring gives the following result:

$$\begin{aligned} \frac{1}{16}\psi_{aln}\psi^a_{gh}\psi_{dmi}\psi_{ejk}(\epsilon^{deln}\epsilon^{ghbc})(\epsilon^{mi}{}_b{}^f\epsilon^{jk}{}_c{}_f) &= \psi^a{}_m\psi^{ade}\psi_{dm}{}^l\psi_{eil} - \frac{1}{2}\psi^a{}_a{}^d\psi^e{}_{de}\psi_{mil}\psi^{mil} \\ &+ 2\psi_{ad}{}^m\psi^{ade}\psi_e{}^{il}\psi_{iml} - \frac{1}{2}\psi_{ade}\psi^{ade}\psi^{mil}\psi_{iml} + 2\psi^a{}_a{}^d\psi^e{}_d{}^m\psi_{iml}\psi_e{}^{il} \end{aligned} \quad (22)$$

The two expressions are not identical! Even if one uses the Bianchi identity, the two results do not convert into each other! In finding the couplings in (16) we have used the specific paring that the two Levi-Civita tensors in each Q contract with each other.

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Appendix

In this appendix we review the low energy expansion of the S-matrix element of four gauge bosons in the superstring theory. The S-matrix element has been calculated in [29],

$$\mathcal{A}_{1234} \sim \frac{\Gamma(-s)\Gamma(-t)}{\Gamma(1+u)}K \quad (23)$$

where the Mandelstam variables are $s = -k_1 \cdot k_2$, $t = -k_1 \cdot k_4$, $u = -k_1 \cdot k_3$ which satisfies $s + t + u = 0$, and K is the following kinematic factor:

$$\begin{aligned} K &= -k_1 \cdot k_2 (\zeta_1 \cdot k_4 \zeta_3 \cdot k_2 \zeta_2 \cdot \zeta_4 + \zeta_2 \cdot k_3 \zeta_4 \cdot k_1 \zeta_1 \cdot \zeta_3 + \zeta_1 \cdot k_3 \zeta_4 \cdot k_2 \zeta_2 \cdot \zeta_3 + \zeta_2 \cdot k_4 \zeta_3 \cdot k_1 \zeta_1 \cdot \zeta_4) \\ &- k_2 \cdot k_3 k_2 \cdot k_4 \zeta_1 \cdot \zeta_2 \zeta_3 \cdot \zeta_4 + \{1, 2, 3, 4 \rightarrow 1, 3, 2, 4\} + \{1, 2, 3, 4 \rightarrow 1, 4, 3, 2\} \end{aligned} \quad (24)$$

which is stu symmetric. In above amplitude $\alpha' = 1/2$. The α' -expansion of the Gamma functions is

$$\frac{\Gamma(-s)\Gamma(-t)}{\Gamma(1+u)} = \frac{1}{st} - \frac{\pi^2}{6} - \zeta(3)(s+t) - \frac{\pi^4}{360}(4s^2 + st + 4t^2) + \dots$$

The total amplitude includes all non-cyclic permutation of the external states, *i.e.*,

$$\mathcal{A} = \mathcal{A}_{1234} + \mathcal{A}_{1243} + \mathcal{A}_{1324} + \mathcal{A}_{1342} + \mathcal{A}_{1423} + \mathcal{A}_{1432} \quad (25)$$

Using the relation $s + t + u = 0$, one finds that \mathcal{A} has no massless pole. It becomes

$$\mathcal{A} \sim -\left[\pi^2 + \frac{\pi^4}{24}(t^2 + s^2 + u^2) + \dots\right] K \quad (26)$$

which produces only contact terms with four, eight, and higher momenta. The four momenta terms are reproduced by the DBI action (1) which are proportional to π^2 , and its eight momenta terms are reproduced by Ω^4 terms [12] which are proportional to π^4 . The couplings for the transverse scalars can be found from the above couplings by using the condition that the scalar polarization is in transverse space, *i.e.*, $\zeta_i \cdot k_j = 0$.

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